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Random-phase approximation for the grand-canonical potential of composite fermions in the half-filled lowest Landau level*

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Abstract. We reconsider the theory of the half-filled lowest Landau level using the Chern-Simons formulation and study the grand-canonical potential in the random-phase approximation (RPA). Calculating the unperturbed response functions for current- and charge-density exactly, without any expansion with respect to frequency or wave vector, we find that the integral for the ground-state energy converges rapidly (algebraically) at large wave vectors k, but exhibits a logarithmic divergence at small k. This divergence originates in the k^{-2} singularity of the Chern-Simons interaction and it is already present in lowest-order perturbation theory. A similar divergence appears in the chemical potential. Beyond the RPA, we identify diagrams for the grand-canonical potential (ladder-type, maximally crossed, or a combination of both) which diverge with powers of the logarithm. We expand our result for the RPA ground-state energy in the strength of the Coulomb interaction. The linear term is finite and its value compares well with numerical simulations of interacting electrons in the lowest Landau level.

PACS. 71.10.Pm Fermions in reduced dimensions (anyons, composite fermions, Luttinger liquid, etc.) – 73.40.Hm Quantum Hall effect (integer and fractional) – 71.27.+a Strongly correlated electron systems; heavy fermions

1 Introduction

The combination of an electronic interaction and a strong magnetic field in a two-dimensional electron system yields a rich variety of phases. These are best classified by the filling factor ν, which is the electron density divided by the density of a completely filled Landau level. In the case of $\nu \cong 1/2$, the behavior of the system resembles that of a Fermi liquid in the absence of a magnetic field, or at small magnetic fields, and over the past years, a intriguing picture has emerged: at $\nu = 1/2$, each electron combines with two flux quanta of the magnetic field to form a composite fermion; these composite fermions then move in an effective magnetic field which is zero on the average. The interpretation of many experiments supports this picture. Here, we mention transport experiments with quantum (anti-) dots [1], in which features of the resistivity are related to closed loops of the composite fermions around the dots, and also focusing experiments [2]; more references to recent work can be found in [3–6].

Theoretically, the picture was developed in the pioneering work of Halperin, Lee, and Read [7]. They formulated the Hamiltonian in terms of Chern-Simons (CS) transformed electrons and studied within the random-phase approximation (RPA) many physical quantities. Most prominent among these is the effective mass of the composite fermions which they found to diverge at the Fermi surface. This problem of the effective mass was again taken up by Stern and Halperin [8] who calculated the effective mass and also the quasiparticle interaction function of Landau's Fermi-liquid theory, see also [9,10]. Recently, Kopietz and Castilla [11] used the method of higher-dimensional bosonization to extend the analysis beyond the RPA. Thus treating the infrared singularities, they found that the vertex corrections render the quasiparticle mass finite. The most difficult problem in all these treatments is that the calculated composite fermion spectrum should scale with the strength of the Coulomb repulsion. Most recently, Shankar and Murthy [12] proposed, within the CS approach, a new parameterization which allows to separate the contributions of the CS particles from those of the magneto-plasmon oscillators in the Hamiltonian. After they restrict the number of the magneto-plasmon oscillators to the number of electrons in the original Hamiltonian, they get a finite quasiparticle mass which scales with the inverse of the strength of the Coulomb repulsion.

The CS formulation of interacting electrons, with a density such that the filling factor of a Landau level

* Dedicated to J. Zittartz on the occasion of his 60th birthday

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is $\nu \cong 1/2$, nicely realizes the concept of composite fermions, in accordance with the experimental observations. The price to be paid for using the CS transformation is that the non-interacting system of electrons in a strong magnetic field, after the transformation, becomes a highly complicated, interacting problem – formulated in terms of the CS field. The CS interaction diverges for small wave vector k as $1/k^2$. Therefore, the singular diagrams in leading order need to be resumed; these diagrams are well known as the RPA of the jellium problem in $d = 3$. But there is no small parameter in the problem of non-interacting electrons in a strong magnetic field and there is no reason to expect that the RPA diagrams already reproduce the correct result. In fact, as we shall see, the RPA is even plagued with a marginal divergence in the energy. The understanding, and even the evaluation of the RPA, still seem to be incomplete: Is it necessary to introduce an ultraviolet cut off in the momentum integrals, in order to restrict the number of the magneto-plasmon oscillators, or is that already inherent in the theory? After all, the introduction of the cut off seems to be crucial for the conclusions in the work of Shankar and Murthy. The simplest quantity for a study of this question is the ground-state energy. Its evaluation in RPA should be straight forward. But all approaches, of which we are aware, concentrate on the behavior at small wave vector. Therefore, we reconsider the RPA and calculate the expression for the energy, refraining from any approximations in the bare response functions.

There is a second motivation for studying the energy in RPA. For physical reasons, it is clear that the exact energy per particle should be expandable in the strength of the Coulomb interaction [13]. The lowest-order term is exactly known, of course, it is the energy of the unperturbed lowest Landau level; still, one can not expect to obtain that result (or the corresponding degeneracy) correctly within the RPA. Given the lowest order, we can proceed calculating – in RPA – the next, the term linear in the strength of the Coulomb interaction. Within a restriction to wave functions of the lowest Landau level, there are no corrections beyond this first order. The contribution of the Coulomb interaction to the total energy seems not to be measurable; still, there are numerical simulations of interacting electrons in the lowest Landau level by Morf and d'Ambrumenil [14] and Girlich [15] which are well suited for a comparison with our analytical result. We want to add that such an expansion with respect to the Coulomb interaction should be useful also for the calculation of other quantities, as the self energy of the composite fermions, because there the scale is set by the Coulomb interaction.

The main problem of the RPA, which does not exist for interacting electrons at zero magnetic field in $d = 3$, remains the logarithmic infrared divergencies caused by the $1/k^2$ behavior of the CS interaction at small wave vector. Such a divergence already appears in the lowest order in the RPA. Beyond the RPA, there are diagrams showing divergencies with powers of the logarithm.

The outline of this work is as follows. First, we introduce the model in its CS formulation. Then, we calculate ground-state energy and chemical potential from the grand-canonical potential in RPA using the exact bare response functions. Further, we calculate the first term of the expansion of the energy in the electron-electron (Coulomb) interaction. Finally, we discuss the logarithmically divergent diagrams beyond the RPA.

2 Chern-Simons description

We consider interacting (spin aligned) electrons moving in two dimensions in a strong magnetic field B in the negative z-direction transverse to the system. The electronic density of the system is chosen such that the lowest Landau level of a non-interacting system is filled to a fraction of $\nu = 1/\tilde{\phi}$ where $\tilde{\phi}$ is an even integer. We are mainly interested in $\tilde{\phi} = 2$. Following the work of Halperin, Lee, and Read [7], we describe the Hamiltonian in terms of Chern-Simons (CS) fermions or composite fermions which consist of the original electrons and $\tilde{\phi}$ attached flux quanta:

$$H = \sum_{\mathbf{k}} (\epsilon_k - \mu)\, a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \frac{1}{2F} \sum_{\substack{\mathbf{k}_1,\mathbf{k}_2 \\ \mathbf{k}\neq 0}} W_{\mathbf{k};\,\mathbf{k}_1,\mathbf{k}_2}\; a^\dagger_{\mathbf{k}_1} a^\dagger_{\mathbf{k}_2-\mathbf{k}} a_{\mathbf{k}_2} a_{\mathbf{k}_1-\mathbf{k}} \ . \quad (1)$$

Here, $a^\dagger_{\mathbf{k}}$ creates (and $a_{\mathbf{k}}$ annihilates) a CS fermion with wave vector $\mathbf{k}$. Since we consider only cases where the filling factor is given by $1/\tilde{\phi}$, the sum of the external magnetic field and the mean CS field, ΔB, vanishes (compare Ref. [7]) and ϵ_k is given by the free spectrum $\epsilon_k = k^2/(2m_b)$, where m_b is the electron band mass; we use $\hbar = 1$, but restore $\hbar$ in final formulae. The chemical potential is denoted by μ. Because of $\Delta B = 0$, we can use periodic boundary conditions; F is the area of the sample. The interaction has contributions from the fluctuations of the CS vector potential and from the original Coulomb interaction of the electrons, $V(r) = \epsilon^2/r$, where $\epsilon^2 = e^2/(4\pi\varepsilon_r\varepsilon_0)$ and ε_r is the dielectric constant. Collecting all these contributions, we get

$$W_{\mathbf{k};\,\mathbf{k}_1,\mathbf{k}_2} = W(k) + \frac{2\pi\tilde{\phi}}{m_b}\, i \left((\mathbf{k}_1 - \mathbf{k}_2) \times \frac{\mathbf{k}}{k^2} \right) \mathbf{e}_z ; \quad (2)$$

$\mathbf{e}_z$ denotes the unit vector in z direction, orthogonal to the system. Here, the last term comes from the term linear in the CS vector potential. The quadratic term is contained in $K(k)$ in

$$W(k) = K(k) + V(k) \ , \quad (3)$$
$$K(k) = \frac{(2\pi\tilde{\phi})^2}{m_b} \frac{N}{F} \frac{1}{k^2} \ ,$$
$$V(k) = \frac{2\pi\epsilon^2}{k} \ .$$

Rewriting the original Hamiltonian in terms of CS operators $a^\dagger$, a as in the Hamiltonian (1), we have made an

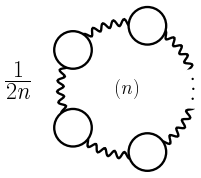
approximation which is already implicit in the treatment of reference [7]: In the term quadratic in the CS vector potential, we have replaced a pair of operators, $a^\dagger a$, by its average and neglected thus the third order fluctuations in the density, which would lead to three particle interactions, *i.e.*, sixth order in $a^\dagger$, a. This average leads to the appearance of the density N/F in the CS interaction $K(k)$; N is the mean electron number, the CS particle number. For the filling factors ν which we consider here, we have

$$\frac{N}{F} = \frac{1}{2\pi l_B^2}\,\frac{1}{\tilde{\phi}}\ . \tag{4}$$

l_B is the magnetic length, $l_B^{-2} = |eB|/\hbar$. The Hamiltonian (1) is the starting point for a standard perturbation theoretical (diagrammatic) treatment. In the following, we will limit ourselves to a study of the grand-canonical potential within the RPA. This will enable us to derive explicit results for the ground-state energy and the chemical potential.

3 Grand-canonical potential

The calculation of the grand-canonical potential $\Omega(T, F, \mu)$ in the RPA is described in text books [16]. The only unusual feature in the present case is the (linear) dependence of the interaction $W_{\mathbf{k};\mathbf{k}_1,\mathbf{k}_2}$ on $\mathbf{k}_1 - \mathbf{k}_2$. Thus, one has to consider two kinds of vertices in the perturbation theory, a density vertex and a current vertex, and the RPA equation becomes a matrix equation as stressed already in reference [7]. The grand-canonical potential in the RPA, Ω_{RPA}, is given by the series of ring diagrams:



$$= -\beta\Omega^{(n)}_{RPA}(T, F, \mu) = \mathcal{O}\left(\left[\frac{1}{\eta^2}\right]^{n-1}\right)\ . \tag{5}$$

Here, we indicate the order of the leading singularity as the infrared wave vector cutoff, $\eta \propto 1/\sqrt{F}$, decreases. For $n = 1$, this is a logarithmic singularity. The singularity originates from the divergence of the CS interaction $K(k)$ at small wave vector. The symmetry factor, $1/(2n)$, is explicitely written in the diagram. We use the Matsubara technique and write the result as

$$\begin{aligned}&\Omega_{RPA}(T, F, \mu) = \\ &-\frac{1}{\beta}\sum_{\mathbf{k}} \ln\left\{1 + e^{-\beta(\epsilon_k - \mu)}\right\} \\ &+\frac{1}{2\beta}\sum_{\mathbf{k}\neq 0}\sum_{\omega} \ln\left\{1 - \Pi_{00}(k,\omega)\left[W(k) + \Pi_{11}(k,\omega)\right]\right\}\ .\end{aligned} \tag{6}$$

Here, the first term is the grand-canonical potential of the free CS fermions. The second term is the contribution of the magneto-plasmon oscillators. In (6), the sum is on Matsubara frequencies of Bose type ($\omega = 2\pi n/\beta$, n integer) and $\beta = 1/(k_B T)$. The response functions of the unperturbed system, Π_{00} and Π_{11}, are defined and studied in the next section.

3.1 Unperturbed response functions

As explained in the introduction, our aim is to evaluate ground-state energy and chemical potential within the RPA without additional approximations. Therefore, we need to calculate the unperturbed response functions exactly. They are given by a one-loop integral involving two unperturbed single-particle Greens functions. The density-density response function (polarization part), Π_{00}, was calculated for the jellium problem in $d = 3$ by Lindhard [17]. In the present case of $d = 2$, fortunately, both response functions can again be calculated analytically. After performing the sum over Matsubara frequencies (of Fermi type), we get

$$\begin{aligned}\Pi_{00}(k,\omega) = &\int \frac{d^2p}{(2\pi)^2}\,\frac{e^{-i\omega 0^+}\ n_F(\epsilon_p - \mu)}{-i\omega + \epsilon_p - \epsilon_{|\mathbf{p}-\mathbf{k}|}} \\ &+(\omega \to -\omega)\end{aligned} \tag{7}$$

and

$$\begin{aligned}\Pi_{11}(k,\omega) = &\left(\frac{2\pi\tilde{\phi}}{m_b}\right)^2 \int \frac{d^2p}{(2\pi)^2}\,\frac{(\mathbf{p}\times\mathbf{k})^2}{k^4} \\ &\times\frac{e^{-i\omega 0^+}\ n_F(\epsilon_p - \mu)}{-i\omega + \epsilon_p - \epsilon_{|\mathbf{p}-\mathbf{k}|}} \\ &+(\omega \to -\omega)\ .\end{aligned} \tag{8}$$

Here, $n_F(x)$ is the Fermi function. The functions Π_{00} and Π_{11} depend only on the absolute value of the wave vector $\mathbf{k}$. Note that in order to simplify our formulae, we absorbed a prefactor in Π_{11}. In the limit of zero temperature, when n_F becomes the step function, it is a tedious, but straight forward task to perform the remaining integrals exactly without any approximative expansion in k or ω. The zero-temperature results are

$$\Pi_{00}(k,\omega) = -\frac{m_b}{4\pi}\ e^{-i\omega 0^+}\ z\left(\frac{\omega}{\epsilon_k}, \frac{k}{2k_F}\right) + (\omega \to -\omega) \tag{9}$$

and

$$\begin{aligned}\Pi_{11}(k,\omega) = &\frac{\pi\tilde{\phi}^2}{48 m_b}\ e^{-i\omega 0^+} \\ &\times\left(z\left(\frac{\omega}{\epsilon_k}, \frac{k}{2k_F}\right)^3 - 3\left(\frac{2k_F}{k}\right)^2 z\left(\frac{\omega}{\epsilon_k}, \frac{k}{2k_F}\right)\right) \\ &+(\omega \to -\omega)\ ,\end{aligned} \tag{10}$$

where k_F is the Fermi wave vector defined by $\mu = k_F^2/(2m_b)$. We use the abbreviation

$$z(x,u) = 1 + ix - \sqrt{(1+ix)^2 - u^{-2}}\ , \tag{11}$$

and we define the square root as analytical continuation from positive values of the root for positive radicands. Our results, the zero-temperature response functions Π_{00} and Π_{11}, are correct for arbitrary large or small values of ω/ϵ_k and $k/(2k_F)$. We still keep the convergence factor $\exp(-i\omega 0^+)$, because that will be crucial in the following, where we have to perform the frequency integral over Π_{00} ($\sim 1/\omega$ for large ω).

3.2 Ground-state energy

The ground-state energy coincides with the grand-canonical potential for $T \to 0$ after adding the term $N\mu$. Using the expression (6), we get (here, $n_F(x)$ denotes the Fermi function at zero temperature)

$$\begin{aligned} E_{RPA} &= \sum_{\mathbf{k}} n_F(\epsilon_k - \mu)\, \epsilon_k \\ &+ \frac{\hbar}{2} \sum_{\mathbf{k}\neq 0} \int_0^\infty \frac{d\omega}{\pi} \ln\{1 - \Pi_{00}(k,\omega)[W(k) + \Pi_{11}(k,\omega)]\} \\ &= \sum_{\mathbf{k}} n_F(\epsilon_k - \mu)\, \epsilon_k + \sum_{\mathbf{k}\neq 0} \frac{1}{2}\hbar\omega_c\ \hat{e}\left(\frac{k}{2k_F}, \frac{\hbar\omega_c}{2\mu}\right) \end{aligned} \tag{12}$$

($\hbar\omega_c = \hbar^2/(m_b l_B^2)$). Here, the first term is the energy of the free CS fermions and the second term is the contribution of the magneto-plasmon oscillators. While the first sum is confined to the Fermi sphere, one has to discuss the convergence of the second sum which we will do below. In order to simplify the following analysis, we have defined

$$\hat{e}(u,v) = -\frac{1}{4}\left(\frac{\tilde{\phi}}{2}\frac{1}{u^2} + \frac{\lambda}{uv}\right) + e(u,v)\ , \tag{13}$$

with

$$e(u,v) = \frac{2u^2}{v}\int_0^\infty \frac{dx}{\pi}\ \ln\{1 + I(x,u,v)\} \tag{14}$$

and

$$\begin{aligned} I(x,u,v) = (1-z_+)\Bigg(&\frac{\tilde{\phi}}{2}\frac{v}{2u^2} + \frac{\lambda}{2u} + (\frac{\tilde{\phi}}{2})^2\frac{1}{12}(1-z_+) \\ &\times\left[(1-z_+)^2(1-3z_-^2) - \frac{3}{u^2}\right]\Bigg)\,. \end{aligned} \tag{15}$$

From now on, we will use $\lambda = \epsilon^2 k_F/(2\mu)$ as a dimensionless measure of the Coulomb interaction. $z_+(x,u)$ and $z_-(x,u)$ are related to the real and imaginary parts of $z(x,u)$ in (11):

$$z_\pm(x,u) = \sqrt{\frac{1}{2}\left[\sqrt{(1-u^{-2}-x^2)^2 + 4x^2} \pm (1-u^{-2}-x^2)\right]}\,. \tag{16}$$

The difference between the energy distributions $\hat{e}$ and e is due to the following: For large ω, $\Pi_{00}(k,\omega)$ and $\Pi_{11}(k,\omega)$ decay as ω^{-1}. Therefore, one has to study the convergence of the ω integral in (12). Here, in the first term of an expansion of the $\ln\{\cdots\}$ around 1, the factors $\exp(-i\omega 0^+)$ in $\Pi_{00}(k,\omega)$ define the integration; in the higher terms in the expansion, these convergence factors can be put equal to one. $e(u,v)$ is the expression which one would obtain setting these factors equal to one everywhere. The difference between $\hat{e}$ and e thus comes from the difference in the first term in the expansion of the $\ln\{\cdots\}$, calculated once with and once without convergence factors.

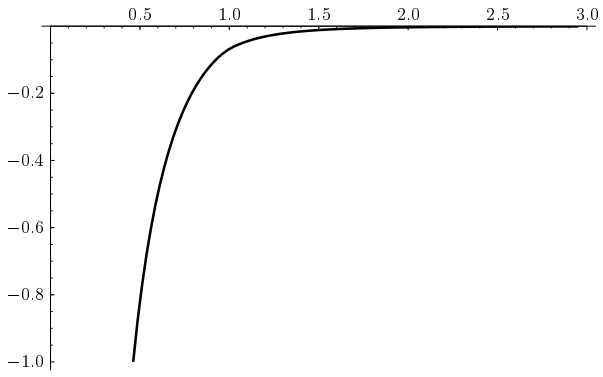


Fig. 1. $\hat{e}(u,1)$ for $\tilde{\phi} = 2$ and $\lambda = 0.7$.

Now, we analyse the contribution of the magneto-plasmon oscillators to E_{RPA} by studying the dependence of $\hat{e}(u,v)$ on $u = k/(2k_F)$.

(i) We first discuss the behavior of $\hat{e}(u,v)$ for large u. With

$$\begin{aligned} z_+(x,u) &= 1 - \frac{1}{2u^2}\frac{1}{1+x^2} - \frac{1}{8u^4}\frac{1-3x^2}{(1+x^2)^3} + \mathcal{O}(u^{-6})\ , \\ z_-(x,u) &= x + \mathcal{O}(u^{-2}) \end{aligned} \tag{17}$$

a straight forward expansion of $e(u,v)$ yields

$$\hat{e}(u,v) = -\frac{1}{2v}\frac{1}{(2u)^4}\left[\left(\frac{\tilde{\phi}}{2}\right)^2 + \frac{1}{2}\lambda^2\right] + \mathcal{O}(u^{-5})\,. \tag{18}$$

Thus, the integral defining E_{RPA} is *convergent for large wave vectors* k. There is no need of an artificial cut off in the ultraviolet. A numerical integration of (14) shows in fact that the cut off is rather abrupt, *cf.* Figure 1. We shall justify below the values $v = 1$ and $\lambda = 0.7$, which we have chosen here for v and the strength of the Coulomb interaction.

(ii) The opposite case of small u leads to a divergence. The integral (14) is finite; its dominant contributions come from $x \propto u^{-2}$. We find for $u = 0$

$$e(0,v) = \sqrt{\frac{1}{v}\frac{\tilde{\phi}}{2}}\,. \tag{19}$$

The result of the numerical integration for general u as shown in Figure 2 confirms a smooth behavior of $e(u,v)$ for small u. The negative curvature in the region near $u = 0.9$ is seen to be related to the value of λ.

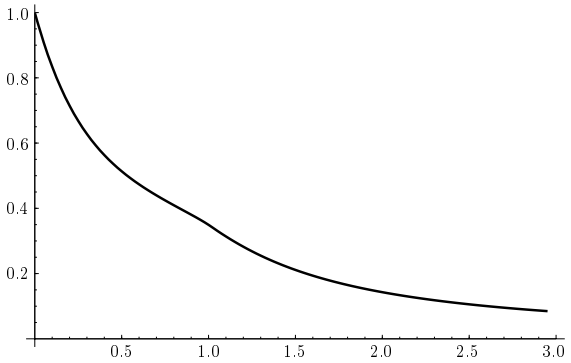


Fig. 2. $e(u,1)$ for $\tilde{\phi}=2$ and $\lambda=0.7$.

This feature gets less pronounced as λ is decreased. Hence, $\hat{e}(u,v) \propto -\tilde{\phi}/(8u^2) \propto k^{-2}$ for small k and E_{RPA} contains *a logarithmic divergence in the infrared regime.* This divergent term in the energy is the same as that found by Halperin, Lee, and Read [7], *cf.* their equation (6.27). It is clear from the derivation, that this logarithmic divergence comes from the first diagram in the RPA series in (5):

$\frac{1}{2}$ 

$$\begin{aligned} &= -\beta\Omega^{(1)}_{RPA} \\ &\sim \frac{\beta}{2F}\sum_{\mathbf{k}\neq 0} K(k) \sum_{\mathbf{k}_1} n_F(\mathbf{k}_1+\mathbf{k}) n_F(\mathbf{k}_1) \\ &\sim N\frac{1}{4v}\tilde{\phi}^2\ \beta\hbar\omega_c\ \ln\frac{\kappa}{\eta}\,. \end{aligned} \tag{20}$$

Here, we give only the term with the leading singularity for decreasing infrared cut off η; $\kappa \ll k_F$ is an upper wave vector cut off. The result (20) is quickly reproduced if one uses only the leading term $\hat{e}(u,v) \sim -\tilde{\phi}/(8u^2)$ in the integral for the energy (12). The logarithmic divergence is explained as the result of the long-range behavior of the CS interaction $K(k)$ in a two-dimensional system. Thus we see, that within the RPA, the ground-state energy is infinite.

We wish to add two remarks. First, this divergence in the energy is due to an insufficient treatment of the system even without electron-electron interaction, *i.e.* the perturbational treatment of the CS interaction. It is independent of the electron-electron interaction, and, therefore the divergence is unrelated to the inverse effective mass which should scale with the strength of the Coulomb interaction. Second, in other cases of RPA calculations, such a divergence in the energy does not appear; in the case of the Coulomb interaction in three dimensions, for example, one has a convergent integral ($\int d^3k/k^2$) and the same is true for the Coulomb interaction in two dimensions. Consequently, for the case of the CS interaction, it becomes necessary to study diagrams *beyond* the RPA.

3.3 Chemical potential

The chemical potential, as the ground-state energy, should be derived from the grand-canonical potential. Calculating the mean particle number N from Ω_{RPA} and using its value given by the filling factor (4), we get at zero temperature

$$v = \frac{\tilde{\phi}}{2} + 2\tilde{\phi}v\int_0^\infty du\ u\left[u\partial_u + 2v\partial_v + \lambda\partial_\lambda\right]\hat{e}(u,v)\,. \tag{21}$$

This equation should determine the chemical potential μ *via* $\mu = \frac{1}{2}\hbar\omega_c/v$. But $\hat{e}(u,v)$ diverges in the infrared regime as $-\tilde{\phi}/(8u^2)$. Therefore, the r.h.s. of (21) becomes infinitely large and there is no positive solution for the parameter v from this equation. Hence, we have to conclude that within the RPA, there is no satisfactory solution for the chemical potential.

3.4 Expansion of the energy in the Coulomb interaction

Here, we wish to study the expansion of the ground-state energy in the strength of the Coulomb interaction, λ. Generally, we have for the energy per particle

$$E/N = \frac{1}{2}\hbar\omega_c + E^{(1)}/N + \mathcal{O}(\lambda^2)\,. \tag{22}$$

The first term is the energy of the lowest Landau level; in the RPA, it diverges as shown above. Because the CS transformation gives an exact reformulation of the problem also in the absence of an electron-electron interaction, this divergence must be compensated by other divergent diagrams; we expect such a compensation by the diagrams discussed in Section 3.5 below. In the calculation of the second term, now, we encounter the problem that the unperturbed chemical potential can not be determined from the unperturbed grand-canonical potential because the divergence is not eliminated. As a way out, we insert in the second term in (22) the known exact unperturbed value $\mu = \hbar\omega_c/2$. Because for free composite fermions, $\mu = 1/(2m_b)(4\pi N/F)$, *i.e.* $\mu = \hbar\omega_c\nu$ (use Eq. (4)) that value of μ coincides for half-filling accidentally with the chemical potential of the free composite fermions. This substitution fixes the variable v, $v=1$. Using this and taking the electron density $\rho \approx 1.5\times 10^{15}$ m^{-2} from the experiments of Kang *et al.* [1] in GaAs ($\varepsilon_r = 12.8$ and $m_b = 0.068\ m_{el}$), one derives for the value of the relative strength of the Coulomb interaction $\lambda \approx 0.7$ which was used in Figures 1 and 2 together with $v=1$. We now obtain for the second term in (22)

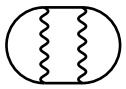
$$E^{(1)}_{RPA}/N = \frac{\epsilon^2}{l_B}\frac{2\tilde{\phi}}{v^{3/2}}\int_0^\infty du\left\{-\frac{1}{4} + u^2\int_0^\infty \frac{dx}{\pi}\frac{1-z_+(x,u)}{1+I_0(x,u,v)}\right\}. \tag{23}$$

Here, $I_0 = I\,|_{\lambda=0}$. A numerical evaluation is simple and the result is $E^{(1)}_{RPA}/N = -0.6\ \epsilon^2/l_B$. This compares

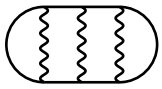
reasonably well with $E^{(1)}_{sim}/N = -0.466\ \epsilon^2/l_B$ obtained in numerical simulations in the spherical geometry by Morf and d'Ambrumenil [14], and by Girlich [15] *via* the threshold energy of the many-particle density of states.

3.5 Higher order logarithmic divergences

In the CS problem at $d = 2$, there are logarithmically divergent diagrams in the grand-canonical potential, because the CS interaction $K(k)$ diverges as $1/k^2$ for small wave vectors. In contrast, in the Coulomb problems at zero magnetic field in $d = 3$ and $d = 2$, such divergences are absent. The lowest-order divergence is found in the logarithmically divergent diagram (20), the first of the RPA diagrams. In addition, there are diagrams for Ω beyond the RPA diverging with higher powers of $\ln \eta$. These diagrams are characterized by independent k_l-integrations over $K(k_l)$; they are either ladder type diagrams or maximally crossed diagrams or combinations of both. The leading singularities of the ladder diagrams in second and third order are

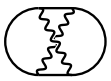
$\frac{1}{2}$ 

$$= -\beta\Omega^{(2)}_{\rm L} \sim \frac{N}{12v}\,\beta\hbar\omega_c\,\tilde{\phi}^4\left(\ln\frac{\kappa}{\eta}\right)^2, \qquad (24)$$

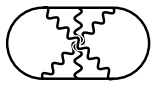
$\frac{1}{2}$ 

$$= -\beta\Omega^{(3)}_{\rm L} \sim \frac{N}{160v^2}\,(\beta\hbar\omega_c)^2\,\tilde{\phi}^6\left(\ln\frac{\kappa}{\eta}\right)^3. \qquad (25)$$

Similarly, for the maximally crossed diagrams

$\frac{1}{2\cdot 2}$ 

$$= -\beta\Omega^{(2)}_{\rm Lx} \sim -\frac{N}{48v}\,\beta\hbar\omega_c\,\tilde{\phi}^4\left(\ln\frac{\kappa}{\eta}\right)^2, \qquad (26)$$

$\frac{1}{2\cdot 3}$ 

$$= -\beta\Omega^{(3)}_{\rm Lx} \sim \frac{N}{144v^2}\,(\beta\hbar\omega_c)^2\,\tilde{\phi}^6\left(\ln\frac{\kappa}{\eta}\right)^3. \qquad (27)$$

Further, insertions of this kind are possible in the ring diagrams (5). The summation of all logarithmically divergent diagrams is expected to give a finite result. This should then lead to a finite ground state energy and a well defined equation for the chemical potential in the absence of an electron-electron interaction. Also, such a summation should improve the result for the Coulomb contribution to the ground-state energy.

4 Conclusion

On the basis of bare response functions calculated analytically for arbitrary k and ω, we evaluated the grand-canonical potential in the RPA without any further approximation. Because of the $1/k^2$-singularity of the Chern-Simons interaction and the system's dimension $d = 2$, the grand-canonical potential contains a logarithmically divergent term. Thus, the ground-state energy and the chemical potential are not well defined within the RPA. On the other hand, the CS transformation leads to the picture of composite fermions forming a Fermi liquid at zero magnetic field, which is experimentally well supported. However, for a system of non-interacting electrons in a strong magnetic field, which we call the "unperturbed problem", this transformation already leads to a very complicated reformulation of the Hamiltonian. We have to conclude that, in the CS formulation, one has to go beyond the RPA in order to get a satisfying solution of that "unperturbed problem".

Further, we calculated the first-order term in the expansion of the ground-state energy with respect to the strength of the Coulomb interaction. There, we used for the chemical potential, which remains undefined in the RPA, the unperturbed value. The result obtained is $\approx$ 25% larger than the result of numerical simulations. In view of the absence of a small parameter in the "unperturbed problem" and given the simple random-phase approximation, this seems to be a reasonable agreement. For an improvement of the calculation of the Coulomb contribution to the energy, the "unperturbed problem" must be solved in a better approximation, which should be free from a divergence.

The "unperturbed problem" is well defined, the CS transformation is a rigorous reformulation; therefore, one should not encounter divergences. Thus, the divergent diagram in the RPA must be compensated by others. We finally studied other logarithmically divergent diagrams, beyond the RPA, and calculated their leading singularities. It is an open question, whether taking these leading singularities into account is already sufficient for a compensation. We consider this compensation, *i.e.* the screening of the CS interaction, to be closely related to the recent discussion of the "dipole-nature" of the composite fermions [18–20]. We plan to return to this question in a following publication.

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